GROUP THINKING STYLES AND THEIR MODELLING PROCESS WHILE ENGAGING IN MODELLING ACTIVITIES

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The current study investigated the relationship between students' thinking style and their modelling process and routes. Thirty-five eighth-grade students were examined. For the first stage, the students solved a word problem, and according to their solutions, they were assigned to one of two groups: a visual thinking style group and an analytic thinking style group. The two groups engaged in three modelling activities. Findings indicating differences in the groups' modelling processes in performing the three activities. The primary differences in the modelling processes were manifested in simplifying, mathematizing, and eliciting a mathematical model. In addition, the analytic thinking group skipped the real-model phase in the three activities, while the visual group built a real model for each activity.

INTRODUCTION

Thinking style and cognitive methods strongly affect student performance in many areas, largely determining significant differences in their performance, as demonstrated in empirical cognitive psychology studies (e.g., Cakan, 2000). Therefore, students' different thinking styles should be taken into account upon determining appropriate educational interventions (Sternberg & Zhang, 2005). Thus, teacher awareness of differential thinking styles is particularly important when students are exposed to modelling activities that offer them the opportunity to meet everyday challenges and demands and provide them with the abilities and competencies to deal with complex systems and real-word situations (Lesh & Doerr, 2003). Mathematical modelling is the process of translating between the real world and mathematics (Blum & Borromeo Ferri, 2009). Knowledge about students' modelling processes can ameliorate their teachers' interventions (Blum & Leiß, 2005). Given their potential, modelling processes have been studied widely (e.g., Blum & Borromeo Ferri, 2009). However, only a few scholars (e.g., Borromeo Ferri, 2010) have examined the modelling process of individuals having different thinking styles. Furthermore, almost no studies have focused on the modelling process with respect to thinking styles characterizing groups, where all modellers in each group have the same thinking style. This study aims to shed light on the influence of group thinking style on their modelling process and modelling route while engaged in modelling activities.

FRAMEWORK

Mathematical thinking Style

A style is a way of thinking; it is not an ability, but rather a preferred way of using one's abilities (Sternberg, 1997). Thus, mathematical thinking styles denote how individuals prefer to learn mathematics, not how their mathematical understanding is assessed. In addition, it also is indicative of how the individual prefers to proceed with the mathematical task (Sternberg, 1997). Klein (cited in Borromeo Ferri & Kaiser, 2003) suggested three different thinking styles: the philosopher, who constructs on the basis of concepts; the analyst, who operates within a formula; and the geometer, who has a visual starting point. Similarly, Borromeo Ferri and Kaiser (2003), in their empirical study, suggested three thinking styles: the analytic, the visual, and the integrated. In the current study, we will follow the latter classification, focusing on the visual and the analytic thinking styles. The visual thinking style has been defined as thinking based on the shapes, drawings, and images presented in real situations and relationships (Campbell, Collis, & Watson, 1995). Students with a visual thinking style are characterized by a strongly image-oriented way of thinking when solving mathematical problems; this facilitates their obtaining, representing, interpreting, perceiving, and memorizing information, as well as expressing it (Borromeo Ferri & Kaiser, 2003). The analytic thinking style has been identified as thinking symbolically and formalistically, involving sorting and teasing out elements from their context. This style reflects a tendency to focus on the properties of objects and elements for classification into categories, preferring to use rules about categories and predicting behavior (Monga & John, 2007).

Modelling

Mathematical modelling means solving complex, realistic, and open problems with the help of mathematics, with the process that students develop and use in solving such problems termed modelling process. The modelling process is a cyclic, in which translating between the real world and mathematics transpires in both directions (Blum & Borromeo Ferri, 2009). The modelling processes from a cognitive perspective identified phases and transitions (Blum & Leiß, 2005). The phases comprise a situation model, a real model, and a mathematical model, mathematical results and real results. The transitions include several actions: understanding the problem and simplifying a situation model; presenting a real model; mathematizing, which leads to constructing a mathematical model; applying mathematical procedures; interpreting the mathematical results; and validating, in which mathematical results are validated in a real-life task. Various visual descriptions of the cyclic process-modelling cycle have been reported in the literature. The current research is based on Blum and Leiß's (2005) modelling cycle. Delineating the modelling process in detail, incorporating the various phases of the modelling cycle on an internal and external level, is referred to as the modelling route (Borromeo Ferri, 2007).

RESEARCH AIM AND QUESTION:

Do and how groups of students with different thinking styles (visual or analytic) differ in their modelling process and their modelling routes while working on a sequence of modelling activities?

METHOD

Research participants and procedure

For the first stage of the study, a questionnaire for identifying participants' thinking style was administered to 35 students in an eighth-grade class. Based on the styles reflected in solving the questionnaire's tasks, students were then classified into three thinking style groups: analytic (14 students), visual (11 students), and integrated (10 students) thinking style groups. As the focus in the current study was the analytic and visual thinking style, we divided the students into two groups, based on their shared thinking styles. For each group, we selected five students (totalling 10 participants) with the assistance of their mathematics teacher in order to maximize matching variables (e.g., gender, mathematics abilities, socioeconomic status). Both groups (analytic and visual) were assigned three modelling activities in the course of three weeks, one activity per week. The modelling activities were adapted from the literature (e.g., Blum & Borromeo Ferri, 2009).

Data sources and analysis

The data collected from two sources: Questionnaire and video recordings. Questionnaire: The study questionnaire comprised eight tasks for classifying students according to their thinking style. Some of these tasks were adapted from other studies (e.g., Lowrie & Clements, 2009), and some were designed by the researchers. The selected tasks were characterized by a variety of topic areas and a variety of possible solution strategies. An example the tasks is the Turf Problem (Lowrie & Clements, 2001. P. 86): *A husband and wife wanted to turf their backyard (put grass squares down). Before purchasing the turf, they had a ground pool put in their backyard. The pool was 3m wide and 5m long. Sensibly, they also paved an area 1m wide around the pool. If turf costs $10 per square meter, how much would it have cost to turf the backyard (150 m² in total) once the pool and the paving were finished*.

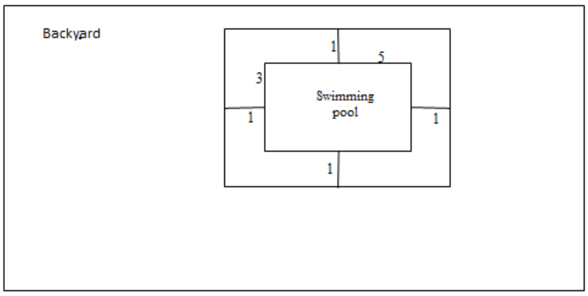
Video recordings: Video recording were made of the two groups working on the three modelling activities and were transcribed.

Questionnaire analysis: We used the constant comparative method (Glaser & Strauss, 1967) to analyze the problem-solving processes for each task in the questionnaire for each student. We adopted the categories described by Borromeo Ferri and Kaiser (2003): When illustrating and solving the mathematical problems, the visual thinking group was characterized by sketches, drawings, or graphs, while the analytical thinking style was expressed in a formula-oriented way that means that information from the text of a given problem, is expressed by means of equations. An example of students' answers classification for the Turf Problem can be seen in Figure 1:

Analytic style: 3+2=5m; 5+2=7m 7 150-35= 115m^2;

7*5=35; 5 115*10= 1150$

Visual style:



5*7=35m^2

150-35=115m^2

115*10=1150$

Figure 1. Samples of students' solutions of the Turf Problem

Video recording analysis: We used the constant comparative method (Glaser & Strauss, 1967) to analyze the students modelling processes in three activities, taking into account the cognitive aspect of modellers' modelling cycle (Blum & Leiß, 2005).

FINDINGS

Modelling process between analytic and visual groups

The findings indicated that the analytic and visual groups demonstrated similar features in working on the three modelling activities, but differed in their modelling processes. Table 1 presents the general findings regarding the two groups' modelling processes.

Table 1: Modelling Processes of the Analytic and Visual Groups in the Three Activities

Group		Analytic									Visual								
Modelling process		Simplifying	Real model	Mathematizing	Mathematical model	Working mathematically	Mathematical results	Interpreting	Real results	Validating	Simplifying	Real model	Mathematizing	Mathematical model	Working mathematically	Mathematical results	Interpreting	Real results	Validating
First activity Modelling cycle	1	-	-	√	√	√	√	-	-	-	√	√	√	√	√	√	-	-	-
	2	-	-	√	√	√	√	√	√	√	-	√	√	√	√	√	√	√	√
	3	-	-	√	√	√	√	√	√	√	-	-	-	-	-	-	-	-	-
Second activity Modelling cycle	1	-	-	-	-	√	√	√	√	-	√	√	√	√	√	√	√	√	√
	2	-	-	-	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	3	-	-	-	√	√	√	√	√	√	-	-	-	-	-	-	-	-	-
Third activity Modelling cycle	1	-	-	√	-	√	√	-	-	√	√	√	√	√	√	√	-	-	√
	2	-	-	√	√	-	-	-	-	-	√	√	√	√	√	√	√	√	√
	3	-	-	√	√	√	√	√	√	√	-	-	-	-	-	-	-	-	-

The analysis of the modelling processes of the two groups in the three activities revealed that the major differences between them were in the real model, simplifying, mathematizing, and mathematical model. Table 2 presents the differences between the two groups, illustrated by sample statements from the students' discussions while working on the problem.

Table 2: Differences in Modelling Process Between Analytic and Visual Groups

Modelling process	Visual group	Analytic group
Simpli-fying	Students seek to illustrate the information in the situations by drawing and illustration. E.g., [5] Student 1: I can explain the situation; we have information about... [they drew illustration of shoes and body]. [6] Student 1: We can find the relation between us and the giants	Students simplified the situations by mathematizing, with skipping real model for the situations. Ex. [5] Student 2: We can calculate by ratio between width and length. [32] Student 3: The ratio between the length of the shoes and height of a person.
Mathe-Matization	Students mathematize the situation by working in tables and lists. E.g., [10] Student 3: Make a table [16] Student 3: Your shoes 26 cm, here I write 26 cm [in the column of the shoes' length] your height is 160.	Students mathematize the situation by searching about formulas. E.g., [9] Student 4: The ratio between the length and the width … length 32 and width 12 [length and width of their shoes]. [11] Student 2: We should simplify the ratio … 32:12.
Mathemati-c	The mathematical model illustrated by	The mathematical model presented

al model	tables and lists	by formula

Modelling cycles and routes in the analytic and visual groups

Analysis of the modelling processes of the two groups in the three modelling activities indicated that the analytic group went through more modelling cycles than did the visual group in each activity to obtain the final model, as presented in Table 1. In addition, the analysis indicated that the analytic group engaged in more skipping during the modelling phases than did the visual group. The groups' modelling processes are presented for the giant's shoes activity (Blum & Borromeo Ferri, 2009) only, due to space limitations. The modelling process of the analytic group can be split into three modelling cycles: the first cycle (C1.1, C1.2, C1.3, C1.4), the second cycle (C2.1, C2.B), and the third cycle (C3.1, C3.B, C3.3, C3.C, C3.4, C3.D, C3.5). Table 3 presents the modelling process and Figure 2 illustrates the modelling route of the analytic group.

Table 3: Modelling Process of the Analytic Group in The Giant's Shoes activity

Modelling cycle	Process	Explanation
The first cycle →	C1.1	Understanding the situation, simplifying through mathematizing by think about the relation between the width and the length of shoes 5.29: 2.37
	C1.2	Working mathematically: Find the ratio between the width and the length of one students; 32:12
	C1.3	Mathematical result: The ratio 8:3
	C1.4	Validating: Not helpful in solving the situation
The second - - → cycle	C2.1	Return to the situation, simplifying through mathematizing: Find the ratio between the length of student's shoes and her height.
	C2.B	Mathematical model: The height of person is four times the length of their

		shoes.
The second cycle	C3.1	Return to the situation, simplifying through mathematizing: Find the ratio between the average of their length of their shoes.
	C3.B	Mathematical model: The length of person is five times the length of shoes
	C3.3	Applying the models: 5.29*5
	C3.C	Mathematical results, the height of the giants is 26.45.
	C3.4	Interpreting to reality, it is almost 27 m
	C3.D	Realistic results 27 m
	C3.5	Validating the results in the situation, 27 m

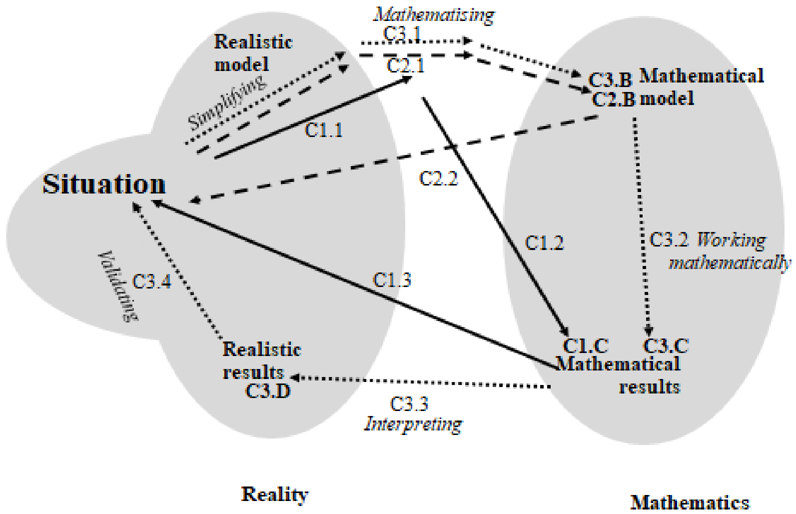


Figure 2: Modelling routes of the analytic group in the Giant's Shoes Activity

The visual group engaged in two modelling cycles: The group began with simplifying the situation through the use of drawing; they tried to draw a figure of shoes through their simplification to yield a real model (A) and thought about the numerical relationship between the giant's height and the length of his shoes, and this relation would be equivalent for ordinary people (C1.1); they began mathematizing by ordering their own shoe length and individuals' height measures, and the ratio between these measurements were recorded on a table they constructed (C1.2); they then elicited a mathematical model, indicating that the ratio between the length of the shoes and the height resembles the ratio of their own measures (C1.B), applied the results (C1.3), and each student received mathematical results resembling his\her ratio, they received different results because each had a different ratio (C1.C); thus, these results didn't resolve the problem (C1.4). The second cycle began with a mathematical model, comprising the average of the group's ratio calculations (C2.B), they applied it (C2.3) and received numerical results 32 (C2.C); this result was then transformed to a realistic result, indicating the giant's height as 32m (C2.D); they accepted this result (C2.5). Figure 3 illustrates the modelling route of the visual group.

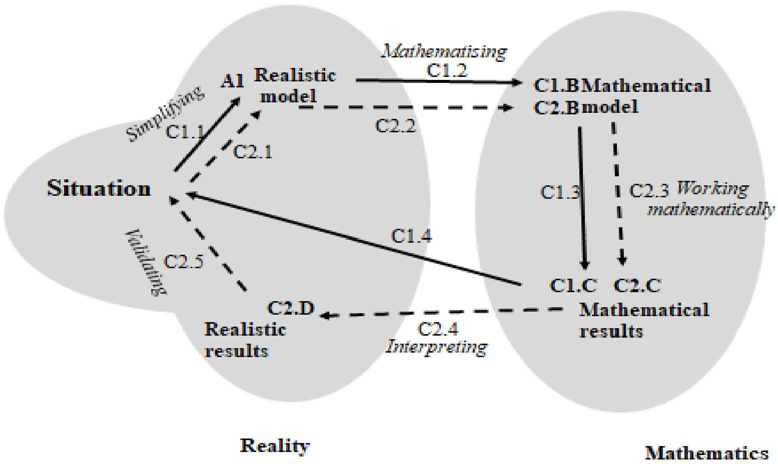


Figure 3: Modelling cycle of the visual group in the Giant's Shoes activity

DISCUSSION

The current study examined the modelling process and routes of two groups of eighth-grade students, an analytic thinking style group and a visual thinking style group. The findings revealed major differences in the two groups' modelling processes. The analytic group tried to simplify the three activities by mathematizing them, while the visual group tried to simplify the activities by drawing and illustrating. In addition, the findings revealed differences in the mathematizing process and in the illustration of the mathematical model. Upon examining the features of the process characterizing the analytic group when engaging in modelling activities, they were found to be similar to features activated in solving routine word problems as Klein (cited in Borromeo Ferri & Kaiser, 2003) reported that students having an analytic thinking style were more likely to search for structures, patterns, or formulas and their application, or briefly operate with formulas. According to the modelling cycle we identified that analytic group engaged in more skipping of modelling phases: In the three activities, they skipped the real model, while the visual group addressed this phase. These findings supported Borromeo Ferri's (2012) findings, she indicated that when analytic thinkers deal with modelling tasks, they preferred to change the real-world situation to a mathematical model and operated in a formalistic manner, while visual thinkers thought more in terms of the real world rather than of formal solutions, thus tending to present their thinking by means of pictures and drawings.

Finally, teachers' awareness of students' thinking styles can have an important role in designing effective interventions. We suggest expanding our work by examining more than a single group from each style in order to collect more information about modelling processes and routes of students with different thinking styles.

REFERENCES

Blum, W., & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt?. *Journal of mathematical modelling and application, 1*(1), 45-58.

Blum, W., & Leiß, D. (2005). "Filling Up"-the problem of independence-preserving teacher interventions in lessons with demanding modelling tasks. In M. Bosch (Ed.), Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education (CERME 4) (pp. 1623-1633). Sant Feliu de Guíxols, Spain: Fundemi Iqs, Universitat Ramon Llull.

Borromeo Ferri, R. (2007). Modelling problems from a cognitive perspective. In C. Haines et al. (Eds.), Mathematical modelling: education, engineering and economics (pp. 260–270). Chichester: Horwood.

Borromeo Ferri, R. (2010). On the influence of mathematical thinking styles on learners modelling behavior. *Journal fur Mathematik- Didaktik, 31*(1), 99-118.

Borromeo Ferri, R., & Kaiser, G. (2003). First Results of a Study of Different Mathematical Thinking Styles of Schoolchildren. In Leone Burton (Ed.) Which Way in Social Justice in Mathematics Education? (pp. 209-239). London: Greenwood

Campbell, K. J., Collis, K. F., & Watson, J. M. (1995). Visual processing during mathematical problem solving. *Educational Studies in Mathematics, 28*(2), 177-194.

Cakan, M. (2000). Interaction between cognitive styles and assessment approaches. Unpublished doctoral dissertation, Louisiana State University, USA. Baton Rouge.

Lesh, R., & Doerr, H.M. (2003). Foundations of models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H.M. Doerr (Eds.), Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching (pp. 3–33). Mahwah, NJ: Erlbaum.

Glaser, B. G., & Strauss, A. L. (1967). Discovery of grounded theory: Strategies for qualitative research. Chicago: Aldine.

Lowrie & M. A. & Clements, K. (2001). Visual and nonvisual processes in grade 6 students' mathematical problem solving. *Journal of Research in Childhood Education, 16*(1), 77-93,

Monga, A. & John, D. (2007). Cultural Differences in Brand Extension Evaluation: The Influence of Analytic versus Holistic Thinking. *Journal of consumer research*, *33*(4), 529- 536.

Sternberg, R. J. (1997). Thinking styles. New York: Cambridge University Press.

Sternberg, R. J., & Zhang, L. F. (2005). Styles of thinking as a basis of differentiated instruction. *Theory into practice, 44*(3), 245-253.